

Exhibit A

ability of a deformed material body to return to its original shape and size when the forces causing the deformation are removed. A body with this ability is said to behave (or respond) elastically.

To a greater or lesser extent, most **solid** materials exhibit elastic behaviour, but there is a limit to the magnitude of the force and the accompanying deformation within which **elastic recovery** is possible for any given material. This limit, called the **elastic limit**, is the maximum stress or force per unit area within a solid material that can arise before the onset of permanent deformation. Stresses beyond the elastic limit cause a material to yield or flow. For such materials the elastic limit marks the end of elastic behaviour and the beginning of plastic behaviour. For most brittle materials, stresses beyond the elastic limit result in fracture with almost no **plastic deformation**.

The elastic limit depends markedly on the type of solid considered; for example, a **steel** bar or wire can be extended elastically only about 1 percent of its original length, while for strips of certain rubberlike materials, elastic extensions of up to 1,000 percent can be achieved. Steel is much stronger than rubber, however, because the tensile force required to effect the maximum elastic extension in rubber is less (by a factor of about 0.01) than that required for steel. The elastic properties of many solids in tension lie between these two extremes.

The different macroscopic elastic properties of steel and rubber result from their very different microscopic structures. The elasticity of steel and other **metals** arises from short-range interatomic forces that, when the material is unstressed, maintain the atoms in regular patterns. Under stress the atomic bonding can be broken at quite small deformations. By contrast, at the microscopic level, rubberlike materials and other **polymers** consist of long-chain molecules that uncoil as the material is extended and recoil in elastic recovery. The mathematical theory of elasticity and its application to engineering mechanics is concerned with the macroscopic response of the material and not with the underlying mechanism that causes it.

In a simple tension test, the elastic response of materials such as steel and bone is typified by a linear relationship between the **tensile stress** (tension or stretching force per unit area of **cross section** of the material), σ , and the extension ratio (difference between extended and initial lengths divided by the initial length), e . In other words, σ is proportional to e ; this is expressed $\sigma = Ee$, where E , the constant of proportionality, is called **Young's modulus**. The value of E depends on the material; the ratio of its values for steel and rubber is about 100,000. The equation $\sigma = Ee$ is known as **Hooke's law** and is an example of a constitutive law. It expresses, in terms of macroscopic quantities, something about the nature (or constitution) of the material. Hooke's law applies essentially to one-dimensional deformations, but it can be extended to more general (three-dimensional) deformations by the introduction of linearly related stresses and strains (generalizations of σ and e) that account for shearing, twisting, and volume changes. The resulting generalized Hooke's law, upon which the linear theory of elasticity is based, provides a good description of the elastic properties of all materials, provided that the deformations correspond to extensions not exceeding about 5 percent. This theory is commonly applied in the analysis of engineering structures and of seismic disturbances.

The elastic limit is in principle different from the **proportional limit**, which marks the end of the kind of elastic behaviour that can be described by Hooke's law, namely, that in which the stress is proportional to the strain (relative deformation) or equivalently that in which the load is proportional to the displacement. The elastic limit nearly coincides with the proportional limit for some elastic materials, so that at times the two are not distinguished; whereas for other materials a region of nonproportional elasticity exists between the two.

The linear theory of elasticity is not adequate for the description of the large deformations that can occur in rubber or in soft human tissue such as skin. The elastic response of these materials is nonlinear except for very small deformations and, for simple tension, can be represented by the constitutive law $\sigma = f(e)$, where $f(e)$ is a mathematical function of e that depends on the material and that approximates to Ee when e is very small. The term nonlinear means that the graph of σ plotted against e is not a straight line, by contrast with the situation in the linear theory. The energy, $W(e)$, stored in the material under the action of the stress σ represents the area under the graph of $\sigma = f(e)$. It is available for transfer into other forms of energy—for example, into the **kinetic energy** of a projectile from a catapult.

The stored-energy function $W(e)$ can be determined by comparing the theoretical relation between σ and e with the results of

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**Main***physics*

law of **elasticity** discovered by the English scientist **Robert Hooke** in 1660, which states that, for relatively small **deformations** of an object, the **displacement** or size of the deformation is directly proportional to the deforming force or load. Under these conditions the object returns to its original shape and size upon removal of the load. Elastic behaviour of solids according to Hooke's law can be explained by the fact that small displacements of their constituent molecules, atoms, or ions from normal positions is also proportional to the force that causes the displacement.

The deforming force may be applied to a solid by stretching, compressing, squeezing, bending, or twisting. Thus a metal wire exhibits elastic behaviour according to Hooke's law because the small increase in its length when stretched by an applied force doubles each time the force is doubled. Mathematically, Hooke's law states that the applied force F equals a constant k times the displacement or change in length x , or $F = kx$. The value of k depends not only on the kind of elastic material under consideration but also on its dimensions and shape.

At relatively large values of applied force, the deformation of the elastic material is often larger than expected on the basis of Hooke's law, even though the material remains elastic and returns to its original shape and size after removal of the force. Hooke's law describes the elastic properties of materials only in the range in which the force and displacement are proportional. (See **deformation and flow**.) Sometimes Hooke's law is formulated as $F = -kx$. In this expression F no longer means the applied force but rather the equal and oppositely directed **restoring force** that causes elastic materials to return to their original dimensions.

Hooke's law may also be expressed in terms of **stress** and **strain**. Stress is the force on unit areas within a material that develops as a result of the externally applied force. Strain is the relative deformation produced by stress. For relatively small stresses, stress is proportional to strain. For particular expressions of Hooke's law in this form, see **bulk modulus**; **shear modulus**; **Young's modulus**.

Related Articles

Aspects of the topic Hookes-law are discussed in the following places at Britannica.

Assorted References

- **bulk modulus** (*in* **bulk modulus (physics)**)

When the bulk modulus is constant (independent of pressure), this is a specific form of Hooke's law of elasticity.

- **discovery** (*in* **Robert Hooke (British scientist)**)

English physicist who discovered the law of elasticity, known as Hooke's law, and who did research in a remarkable variety of fields.

- **elasticity** (*in* **elasticity (physics)**)

...is called Young's modulus. The value of E depends on the material; the ratio of its values for steel and rubber is about 100,000. The equation $\sigma = Ee$ is known as Hooke's law and is an example of a constitutive law. It expresses, in terms of macroscopic quantities, something about the nature (or constitution) of the material. Hooke's law applies essentially to...

- **shear modulus** (*in* **shear modulus (physics)**)